C8229: Quantum Computing

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Stanford Splash Quantum Computing
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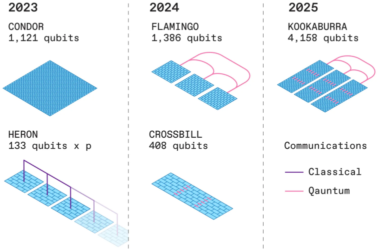
November 15, 2025

Introduction to Quantum Computing

(a) IBM Quantum Computer



(b) Current Machines & Future Plans

- Many believe quantum advantage is about to be reached
- Quantum resources remain expensive

The Quantum Advantage

Certain tasks can be done much better on quantum computers:

- Factorization
- Some types of machine learning
- Optimization
- Data Analysis

Classical Information	Quantum Information
Probabilistic values come from ignorance.	Probabilistic nature is intrinsic.

Probability

To understand quantum computing we need to make sure we're on the same page when it comes to states and probability.

Flipping a Coin

What are the two outcomes?

Probability

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Flipping a Coin

What are the two outcomes?

1. Heads
2. Tails

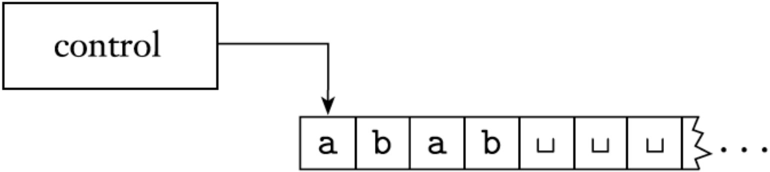
Kets

$$|heads\rangle \quad |tails\rangle$$

A Classical Computer

Church-Turing Thesis

A computer is a *Turing machine*: a tape, a read/write head, and a logical controler.



- The "control" is made out of logic gates!

Classical bits

You probably know that computers operate in binary:

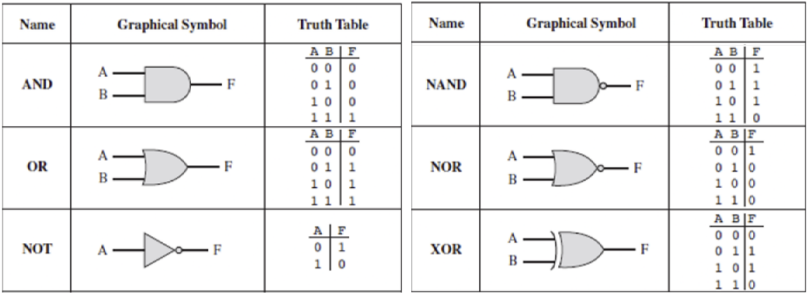
A Bit

A bit can be either 0 or 1.

0

1

Classical Logic Gates

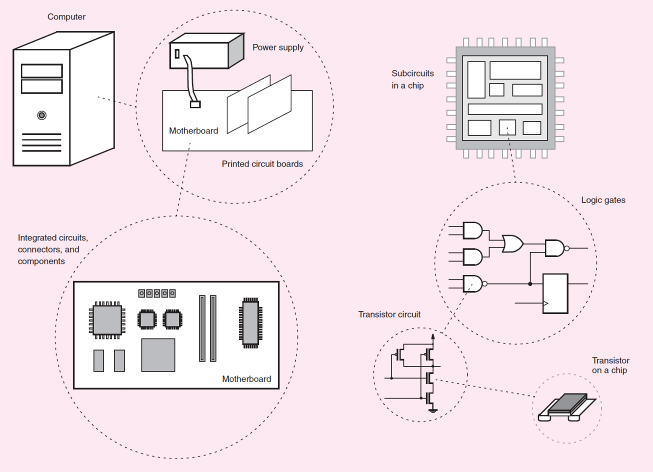


Name	Graphical Symbol	Truth Table
AND	A, B → F	A B | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR	A, B → F	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT	A → F	A | F 0 | 1 1 | 0

Name	Graphical Symbol	Truth Table
NAND	A, B → F	A B | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR	A, B → F	A B | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
XOR	A, B → F	A B | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

Demos: `logic.ly`

How do we build a computer?



Leap to Quantum

In Quantum Computers, we don't use bits but rather qubits. A qubit can be a linear combination of the basis states, $|0\rangle$ and $|1\rangle$. However, a qubit can take more states than just its basis:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

α and β are complex numbers.

Born's Rule

The probability of getting $|0\rangle$ is $|\alpha|^2$. The probability of getting $|1\rangle$ is $|\beta|^2$.

Note: $|a + bi|^2 = (a + bi)(a - bi)$

Discrete Outcomes

The coin is either in $|heads\rangle$ or $|tails\rangle$, but quantum states can be in superposition (indeterminate).
$\psi = \alpha|0\rangle + \beta|1\rangle$ is not determined. However, once you measure it you MUST get either $|0\rangle$ or $|1\rangle$.

Stern Gerlach Experiment

Spin

Electrons have spin. $|\uparrow\rangle$ or $|\downarrow\rangle$, so $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$.

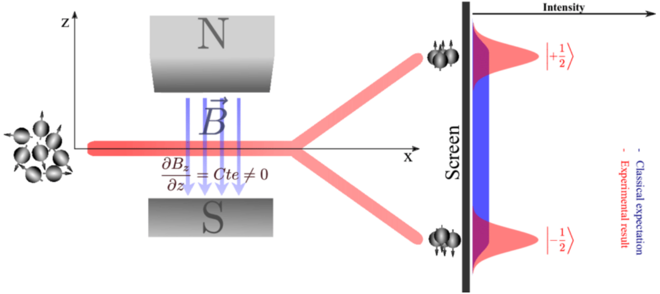


Figure 2: Stern Gerlach Experiment (Saideh, Entanglement of Quantum Systems)

Bloch Sphere

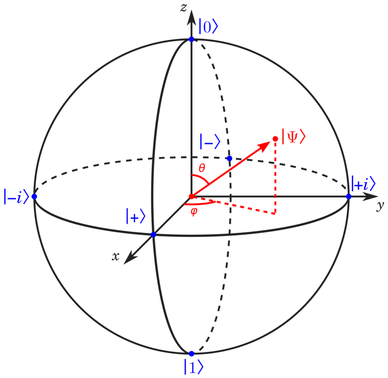


Figure 3: Bloch Sphere

(Fun fact: Felix Bloch was a professor here at Stanford!)

Exercise!

Draw a circle on your board (it's a little hard to work in 3d so let's just talk about real numbers). Label the x and z axis. Draw the following qubit vectors:

$$\psi_1 = |0\rangle \, .$$

$$\psi_2 = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle$$

$$\psi_3 = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle$$

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We call ψ_2 and ψ_3 the x-basis! $|+\rangle \, , |-\rangle$

Classical Gates

Not Gate

$$0 \rightarrow$$

Classical Gates

Not Gate

$$0 \rightarrow 1$$

$$1 \rightarrow$$

Classical Gates

Not Gate

$$0 \rightarrow 1$$

$$1 \rightarrow 0$$

A Quantum Gate

Not Gate / X-Gate / Bit Flip

$$|0\rangle \to |1\rangle, |1\rangle \to |0\rangle.$$

Why is it called the X-Gate?

A Quantum Gate

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$$X \cdot |+\rangle = X \cdot \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{|1\rangle + |0\rangle}{\sqrt{2}} = |+\rangle.$$

It leaves the positive X basis vector unchanged!

$$X \cdot |-\rangle =$$

A Quantum Gate

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$$X \cdot |-\rangle = X \cdot \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{|1\rangle - |0\rangle}{\sqrt{2}} = -|-\rangle.$$

Z Gate

With this in mind, what should the Z gate do if we're following the same pattern?
(Remember, the Z basis is $|0\rangle$ and $|1\rangle$).

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(Remember, the Z basis is $|0\rangle$ and $|1\rangle$).

$$|0\rangle \to |0\rangle$$

$$|1\rangle \to -|1\rangle$$

What happens to $|+\rangle$ and $|-\rangle$?

$$Z \cdot |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = |-\rangle .$$

$$Z \cdot |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}} = \frac{|0\rangle + |1\rangle}{\sqrt{2}} = |+\rangle .$$

Bloch by Bloch

Look back at your Bloch sphere (or draw it again). What does applying an X or Z gate do to a qubit?

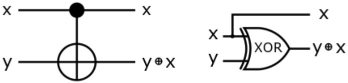
The Hadamard Gate

$$H|0\rangle = |+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$H|1\rangle = |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

Quantum Entanglement

Suppose we have two qubits and a new gate, a CNOT (controlled NOT gate).



input		output	
x	y	x	y+x
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

input		output	
x	y	x	y+x
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Figure 4: CNOT

Now, the two qubits are *entangled*. Why?

Quantum Entanglement Clarified

Suppose I have two envelopes. I've put a Twix in one, and a chocolate coin in another. I send one envelope to person A, and another to person B. Each envelope is therefore in the state:

$$\psi_1 = \psi_2 = \frac{1}{\sqrt{2}} |M\&M\rangle + \frac{1}{\sqrt{2}} |KitKat\rangle$$

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When we "measure" person A's envelope what's the new state?

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When we "measure" person A's envelope what's the new state? It collapsed to $|M\&M\rangle$.

Quantum Entanglement Clarified

1. What state is Person B's envelope in?
2. How long after Person A opened their envelope did the state of Person B's envelope change?
3. Is this change faster than light?

Entangled Qubits

Notation for Multiple Qubits

$$|01\rangle$$

Qubit 1 is in state $|0\rangle$, Qubit 2 is in state $|1\rangle$.

Time for a demo! `https://quantum.ibm.com/composer/`

Construct the Bell Minus State

We want an equal probability of getting $|01\rangle$ and $|10\rangle$ when measuring this state, but we want $|10\rangle$ to be out of phase. Let's construct:

$$\left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}\left|01\right\rangle - \frac{1}{\sqrt{2}}\left|10\right\rangle$$

Hints to be revealed:

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Hints to be revealed:

1. Start with a Hadamard gate on Qubit 0
2. Try using a CNOT gate
3. Try using an X and Z gate

W-State

This time, instead of getting just 2 qubits entangled, let's do 3, with an equal probability of getting each.

1. Initialize our 3 qubit circuit
2. Perform an R_y rotation on qubit 0 by 1.910633 (radians). (See if you can guess what this number is!)
3. Perform a controlled Hadamard gate on qubit 1, with control qubit 0
4. Add a CNOT gate with control qubit 1 and target qubit 2
5. Add a CNOT gate with control qubit 0 and target qubit 1
6. Add a X ($\bigoplus$) gate on qubit 0

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That number was $\cos^{-1} \frac{1}{3}$:)

Heisenberg Uncertainty Principle

One can never perfectly know/measure all 3 angular momenta components of a particle.

No-Cloning Theorem

One can never produce a perfect copy of a quantum state.

Quantum Teleportation Algorithm

We can however, teleport a quantum state (given two entangled qubits).

Assemble a circuit with 3 qubits.

Quantum Teleportation

1. Apply a Hadamard gate to qubit 2.
2. Apply a CNOT gate (control 1 target 2)
3. Phase disk for your convenience
4. CNOT (control 0 target 1)
5. Hadamard qubit 0
6. Measure Qubit 0
7. Measure Qubit 1
8. Controlled RX (π, control 1 target 2)
9. Controlled RZ (π, control 0 target 2)

Test it out! What does it do? How can you tell it works?

Other Interesting Circuits

- Shor's Algorithm - Factorization
  - Why is this useful?
- Variational Quantum Eigensolver

Future Steps

- Learn Qiskit!
- Stanford Quantum High School
- Qubit x Qubit

Acknowledgements

- Thanks y'all!

References

[1] Jack D Hidary. *Quantum Computing: An Applied Approach*. Springer Nature, 2021.

[2] Michael A Nielsen and Isaac L Chuang. *Quantum computation and quantum information*. Cambridge university press, 2010.